Benha University
Faculty of Engineering (Shoubra)
Mathematical Engineering and Physics Department
Final Examination for Qualification Year students

Statistics and probability
Date: 13-6-2015
Time: 2 hours

Answer the following questions

Question (1)(50 marks)

A- A marble is drawn at random from a box containing 10 red, 30 white, 20 blue and 15 orange marbles. Find the probability that it is:
(a) Orange or red,(b) not red or blue, (c) not blue, (d) white, (e) Red, white or blue.

B- If A, B, C are independent events prove that (a) A and BUC, (b) A and B∩C, (c) A and B — C, are independent.

C- A box contains 3 blue and 2 red marbles while another box contains 2 blue and 5 red marbles. A marble drawn at random from one of the boxes turns out to be blue. What is the probability that it came from the first box?

Question (2)(50 marks)

A- Can the function f(x, y) = c(2x+y) be a distribution function? Explain.

B- The joint probability function of two discrete random variables X and Y is given by f(x, y) = cxy for x = 1, 2, 3 and y = 1,2, 3, and equals zero otherwise.
Find (a) the constant c, (b) P(X = 2, У = 3), (c) P($1 \leq X \leq 2, Y \leq 2$), (d) P($X \geq 2$), (e) P(Y < 2), (f) P(X = 1).

C- The mean weight of 500 male students at a certain college is 151 Lb and the standard deviation is 15 Lb. Assuming that the weights are normally distributed, find how many students weigh (a) between 120 and 155 Lb, (b) more than 185 Lb.

Question (3)(50 marks)

A- A bag contains one red and seven white marbles. A marble is drawn from the bag and its color is observed. Then the marble is put back into the bag and the contents are thoroughly mixed. Using (a) the binomial distribution and (b) the Poisson approximation to the binomial distribution, find the probability that in 8 such drawings a red ball is selected exactly 3 times.

B- Suppose that the joint probability function of two discrete random variables X and Y is given by
$$f(x,y) = \begin{cases} c(2x+y) & 2 < x < 6,\ 0 < y < 6 \\ 0 & Otherwise \end{cases}$$
Find (a) the constant c, (b) E(X), (c) $E(Y)$, (d) E(XY), (e) E(X^2) , (f) E(Y^2) , (g) Var(X), (h)Var(y).

C- Find the probability that in family of 4 children there will be (a) at least 1 boy, (b) ath least one boy and at least 1 girl. Assume that the probability of a male birth is ½.

Question (4)(50 marks)

A- If $X^* = (X - \mu)/\sigma$ is a standardized random variable, prove that $E(X^*) = 0$ and $Var(X^*) = 1$.

B- The joint probability function of two discrete random variables X and Y is given by
f$(x,y) = c(2x+y)\ for\ 0 \leq x \leq 2$ and $0 \leq y \leq 3$ and equals zero otherwise.
Find (a) the constant c, (b) E(X), (c) $E(Y)$, (d) E(XY), (e) E(X^2) , (f) E(Y^2) , (g) Var(X), (h) Var(y).

C- Find the probability of getting between 3 and 6 heads inclusive in 10 tosses of fair coin by using (a) The binomial distribution,(b) the normal approximation to the binomial distribution.

Dr. Fathi Abdessalam

Answer of Question (3)

[a]

(a) $$E(X^*) = E\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma}[E(X-\mu)] = \frac{1}{\sigma}[E(X)-\mu] = 0$$

since $E(X) = \mu$.

(b) $$\text{Var}(X^*) = \text{Var}\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma^2}E[(X-\mu)^2] = 1$$

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[b]

(a) $$E(X) = \sum_x \sum_y x f(x,y) = \sum_x x\left[\sum_y f(x,y)\right]$$

$$= (0)(6c) + (1)(14c) + (2)(22c) = 58c = \frac{58}{42} = \frac{29}{21}$$

(b) $$E(Y) = \sum_x \sum_y y f(x,y) = \sum_y y\left[\sum_x f(x,y)\right]$$

$$= (0)(6c) + (1)(9c) + (2)(12c) + (3)(15c) = 78c = \frac{78}{42} = \frac{13}{7}$$

(c) $$E(XY) = \sum_x \sum_y xy\, f(x,y)$$

$$= (0)(0)(0) + (0)(1)(c) + (0)(2)(2c) + (0)(3)(3c)$$

$$+ (1)(0)(2c) + (1)(1)(3c) + (1)(2)(4c) + (1)(3)(5c)$$

$$+ (2)(0)(4c) + (2)(1)(5c) + (2)(2)(6c) + (2)(3)(7c)$$

$$= 102c = \frac{102}{42} = \frac{17}{7}$$

(d) $$E(X^2) = \sum_x \sum_y x^2 f(x,y) = \sum_x x^2\left[\sum_y f(x,y)\right]$$

$$= (0)^2(6c) + (1)^2(14c) + (2)^2(22c) = 102c = \frac{102}{42} = \frac{17}{7}$$

(e) $$E(Y^2) = \sum_x \sum_y y^2 f(x,y) = \sum_y y^2\left[\sum_x f(x,y)\right]$$

$$= (0)^2(6c) + (1)^2(9c) + (2)^2(12c) + (3)^2(15c) = 192c = \frac{192}{42} = \frac{32}{7}$$

(f) $$\sigma_X^2 = \text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{17}{7} - \left(\frac{29}{21}\right)^2 = \frac{230}{441}$$

(g) $$\sigma_Y^2 = \text{Var}(Y) = E(Y^2) - [E(Y)]^2 = \frac{32}{7} - \left(\frac{13}{7}\right)^2 = \frac{55}{49}$$

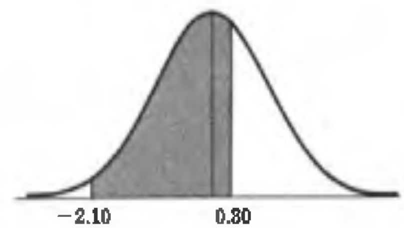
Answer of Question (4)

[a]

(*a*) Weights recorded as being between 120 and 155 lb can actually have any value from 119.5 to 155.5 lb, assuming they are recorded to the nearest pound.

$$119.5 \text{ lb in standard units} = (119.5 - 151)/15 = -2.10$$

$$155.5 \text{ lb in standard units} = (155.5 - 151)/15 = 0.30$$



$$\begin{aligned} \text{Required proportion of students} &= (\text{area between } z = -2.10 \text{ and } z = 0.30) \\ &= (\text{area between } z = -2.10 \text{ and } z = 0) \\ &\quad + (\text{area between } z = 0 \text{ and } z = 0.30) \\ &= 0.4821 + 0.1179 = 0.6000 \end{aligned}$$

Then the number of students weighing between 120 and 155 lb is $500(0.6000) = 300$.

(*b*) Students weighing more than 185 lb must weigh at least 185.5 lb.

$$185.5 \text{ lb in standard units} = (185.5 - 151)/15 = 2.30$$

Required proportion of students

$$\begin{aligned} &= (\text{area to right of } z = 2.30) \\ &= (\text{area to right of } z = 0) \\ &\quad - (\text{area between } z = 0 \text{ and } z = 2.30) \\ &= 0.5 - 0.4893 = 0.0107 \end{aligned}$$

Then the number of students weighing more than 185 lb is $500(0.0107) = 5$.

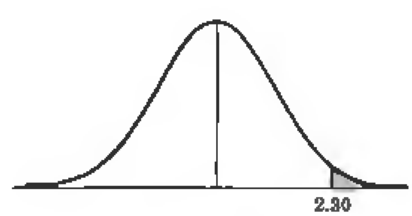


Fig. 4-13

If W denotes the weight of a student chosen at random, we can summarize the above results in terms of probability by writing

$$P(119.5 \leqq W \leqq 155.5) = 0.6000 \qquad P(W \geqq 185.5) = 0.0107$$

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[b]

(*a*) Let X be the random variable giving the number of heads in 10 tosses. Then

$$P(X=3) = \binom{10}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7 = \frac{15}{128} \qquad P(X=4) = \binom{10}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6 = \frac{105}{512}$$

$$P(X=5) = \binom{10}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 = \frac{63}{256} \qquad P(X=6) = \binom{10}{6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 = \frac{105}{512}$$

Then the required probability is

$$P(3 \leqq X \leqq 6) = \frac{15}{128} + \frac{105}{512} + \frac{63}{256} + \frac{105}{512} = \frac{99}{128} = 0.7734$$

(*b*) The probability distribution for the number of heads in 10 tosses of the coin is shown graphically in Figures 4-15 and 4-16, where Fig. 4-16 treats the data as if they were continuous. The required probability is the sum of the areas of the shaded rectangles in Fig. 4-16 and can be approximated by the area under the corresponding normal curve, shown dashed. Treating the data as continuous, it follows that 3 to 6 heads can be considered as 2.5 to 6.5 heads. Also, the mean and variance for the binomial distribution are given by $\mu = np = 10(\frac{1}{2}) = 5$ and $\sigma = \sqrt{npq} = \sqrt{(10)(\frac{1}{2})(\frac{1}{2})} = 1.58$. Now

$$2.5 \text{ in standard units} = (2.5 - 5)/1.58 = -1.58$$

$$6.5 \text{ in standard units} = (6.5 - 5)/1.58 = 0.95$$

Required probability

$= $ (area between $z = -1.58$ and $z = 0.95$)

$= $ (area between $z = -1.58$ and $z = 0$) + (area between $z = 0$ and $z = 0.95$)

$= 0.4429 + 0.3289 = 0.7718$

which compares very well with the true value 0.7734 obtained in part (*a*). The accuracy is even better for larger values of n.

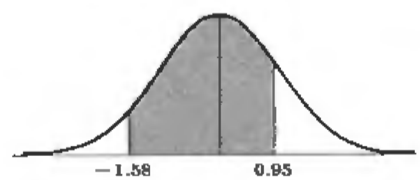

Fig. 4-17

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